Distance-Constrained Satellite Collection as a QUBO Optimization Problem

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January 2026

Contents

1 Abstract

In this paper we work to formulate a satellite collection problem of the first 20 largest satellites by TLE data (gathered on celestrack) as a combinatorial optimization task under mission resource constraints. Given a catalog of orbital objects derived from Two-Line Element (TLE) data, the objective is to maximize the number of satellites that can be collected subject to a total mission cost budget. Mission cost is modeled as a function of distance (or proxy ΔV) between a servicing spacecraft and candidate targets. The resulting problem is a 0–1 travelling salesman problem (TSP), which is NP-hard. We reformulate the constrained optimization problem into a Quadratic Unconstrained Binary Optimization (QUBO) form, enabling solution via quantum annealing or classical QUBO solvers. We attempted to analyze several approaches coming to a conclusion on the best solver to optimize this TSP.

2 Introduction

The rapid growth of orbital debris and inactive satellites in Low Earth Orbit motivates the development of optimized debris collection strategies, especially with the effect of Kessler syndrome exponentially increasing the risk of in orbit collisions. The Kessler syndrome, first described by Kessler and Cour-Palais [1], predicts that collisional cascading in LEO will exponentially increase debris and in turn collisions over time, motivating active retrieval strategies. Work by Olympio and Frouvelle [2] demonstrated that debris selection and guidance can be framed as a combinatorial optimization problem, inspiring our own formulation of satellite retrieval as a QUBO optimization.

Several prior works have addressed related problems. Cerf [3] formulated multi-debris collection as a mixed combinatorial and trajectory optimization problem, establishing early benchmarks for debris selection strategies. More recently, Gagliardi et al. [4] and Swain [5] applied quantum annealing directly to ADR mission planning using QUBO formulations on D-Wave hardware, demonstrating the promise of quantum methods for this domain. Our formulation builds on the Orienteering Problem framework surveyed by Vansteenwegen et al. [8], and uses SGP4 orbital propagation as described by Vallado and Crawford [6]. The application of QUBO to TSP-class problems on quantum annealers is further discussed in [7]

Given limited propulsion resources, a servicing spacecraft must select a subset of candidate targets such that total mission cost does not exceed available budget. This selection process is inherently combinatorial and can be modeled as a binary optimization problem.

In our work, we consider a simplified formulation in which the cost of collecting each object is approximated using distance derived from TLE-based orbital state vectors. The goal is to maximize the number of objects collected within a fixed mission budget.

3 Problem Definition

We consider a satellite retrieval mission in which a servicing spacecraft must collect a subset of high-priority orbital objects under limited mission resources. The objective is to maximize the total importance of retrieved satellites while minimizing total mission cost, modeled through orbital distance.

3.1 Orbital State Representation

Let N denote the number of candidate satellites. Each satellite is represented by its position vector in three-dimensional space, obtained from Two-Line Element (TLE) data using SGP4 propagation:

$$r_i \in \mathbb{R}^3, \quad i = 1, \ldots, N.$$

The servicing spacecraft (depot) is represented by

$$r_0 \in \mathbb{R}^3.$$

3.2 Distance and Cost Model

The cost of traveling between satellites i and j is modeled as the Euclidean distance between their position vectors:

$$d_{ij} = \|r_i - r_j\|_2.$$

Similarly, the cost of traveling between the depot and satellite i is

$$d_{0i} = \|r_0 - r_i\|_2.$$

These distances serve as a proxy for orbital transfer cost (e.g., ΔV).
Each satellite is also assigned an importance weight

$$w_i \geq 0,$$

representing its priority for retrieval.

It is also important to recognize that euclidean distance does not fully represent cost/benefit in this situation. In reality we would need to consider several factors from inclination, phasing, etc. We make this simplification for ease within a combinatorial framework.

3.3 Decision Variables

We consider a permutation (tour) over the set of candidate satellites:

$$\pi = (\pi_1, \pi_2, \ldots, \pi_N),$$

where each satellite appears at most once. The formulation allows for satellites to be effectively skipped through the optimization objective, rather than explicitly constructing variable-length tours.

3.4 Objective Function

The total mission cost of a tour π is defined as

$$C(\pi) = d_{0,\pi_1} + \sum_{j=1}^{k-1} d_{\pi_j,\pi_{j+1}} + d_{\pi_k,0}.$$

The total importance of retrieved satellites is

$$W(\pi) = \sum_{j=1}^{k} w_{\pi_j}.$$

The optimization problem is therefore

$$\max_{\pi} \quad W(\pi)$$

subject to

$$C(\pi) \leq B,$$

where B is the total mission budget.

4 QUBO Formulation

This formulation corresponds to a prize-collecting variant of the Traveling Salesman Problem (TSP), commonly referred to as the Orienteering Problem. It combines aspects of subset selection and route optimization and is known to be NP-hard. Our aspirations are to apply quantum computing techniques, in particular QUBO, to attempt to obtain accurate approximate solutions to this increasingly complex problem.

To enable solution via quantum annealing and classical QUBO solvers, we reformulate the problem as a Quadratic Unconstrained Binary Optimization (QUBO).

4.1 Binary Decision Variables

We introduce time-indexed binary variables:

$$x_{i,t} \in \{0, 1\},$$

where $x_{i,t} = 1$ indicates that satellite i is visited at position t in the tour, for $i = 1, \ldots, N$ and $t = 1, \ldots, N$.

4.2 Routing Cost

The total travel cost is modeled as:

$$\sum_{t=1}^{N-1} \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij}\, x_{i,t} x_{j,t+1} + \sum_{i=1}^{N} d_{0i} x_{i,1} + \sum_{i=1}^{N} d_{i0} x_{i,N}.$$

This captures transitions between consecutive satellites as well as travel to and from the depot.

4.3 Tour Constraints

To enforce a valid tour, we impose the following constraints:

Each satellite is visited exactly once

$$\sum_{t=1}^{N} x_{i,t} = 1 \quad \forall i$$

Each position in the tour is occupied

$$\sum_{i=1}^{N} x_{i,t} = 1 \quad \forall t$$

These constraints are incorporated into the QUBO objective using quadratic penalty terms:

$$\lambda_1 \sum_{i=1}^{N} \left(\sum_{t=1}^{N} x_{i,t} - 1 \right)^2 + \lambda_2 \sum_{t=1}^{N} \left(\sum_{i=1}^{N} x_{i,t} - 1 \right)^2,$$

where $\lambda_1, \lambda_2 > 0$ are penalty parameters.

4.4 Reward Term

To incorporate satellite importance, we include a reward term:

$$-\beta \sum_{i=1}^{N} \sum_{t=1}^{N} w_i x_{i,t},$$

where $\beta > 0$ controls the trade-off between maximizing reward and minimizing cost.

4.5 Final QUBO Formulation

Combining all components, the QUBO formulation is:

$$\min_{x \in \{0,1\}^{N \times N}} \left[\sum_{t=1}^{N-1} \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} x_{i,t} x_{j,t+1} + \sum_{i=1}^{N} d_{0i} x_{i,1} + \sum_{i=1}^{N} d_{i0} x_{i,N} \right.$$
$$\left. + \lambda_1 \sum_{i=1}^{N} \left(\sum_{t=1}^{N} x_{i,t} - 1 \right)^2 + \lambda_2 \sum_{t=1}^{N} \left(\sum_{i=1}^{N} x_{i,t} - 1 \right)^2 - \beta \sum_{i=1}^{N} \sum_{t=1}^{N} w_i x_{i,t} \right].$$

5 Classical Solvers

To evaluate the performance of quantum optimization approaches, we first solve the problem using classical methods to establish baseline results. These methods provide a point of comparison in terms of solution quality and computational efficiency.

5.1 Implementation

We implemented classical solution methods in Python using orbital data derived from TLE inputs. Satellite positions were propagated using the SGP4 model, and pairwise distances were computed to form a cost matrix.

The optimization problem was then solved using classical techniques, including greedy search methods and simulated annealing applied to the resulting distance matrix. The implementation is available in our public repository.[1]

5.2 Solution Approach

Due to the combinatorial complexity of the problem, exact methods become infeasible for large N. Therefore, we employ heuristic approaches to obtain approximate solutions.

In particular, we utilize a greedy nearest-neighbor heuristic, which iteratively selects the next satellite minimizing incremental travel cost. Additionally, we explore stochastic optimization techniques such as simulated annealing to improve upon initial solutions and escape local minima.

The results obtained from these classical methods are used as a baseline for comparison with quantum annealing approaches in subsequent sections.

5.3 Results

N	Method	Cost	Reward	Time (s)
5	Greedy	178,993.05	44.50	0.0011
5	SA	185,309.10	44.50	2.33
10	Greedy	296,958.02	75.85	0.0112
10	SA	262,627.20	75.85	5.51
15	Greedy	311,959.95	120.44	0.0995
15	SA	262,670.08	120.44	9.36
20	Greedy	365,749.15	125.45	0.2479
20	SA	282,554.14	125.45	19.22

Table 1: Comparison of classical solution methods across varying problem sizes. Simulated annealing (SA) results are averaged over multiple runs.

Results Discussion The results demonstrate a clear trade-off between computational efficiency and solution quality. The greedy heuristic produces solutions extremely quickly, with runtimes on the order of milliseconds even for larger problem sizes. However, its solution quality degrades as N increases, resulting in higher total mission cost.

In contrast, simulated annealing consistently produces lower-cost solutions for $N \geq 10$, indicating its ability to escape local minima and improve route optimization. This improvement comes at the expense of significantly higher computational time, with runtimes increasing to several seconds for larger instances.

Interestingly, both methods achieve identical reward values, suggesting that the primary difference lies in route optimization rather than satellite selection. These results establish a strong classical baseline for comparison with quantum annealing approaches.

[1] `https://github.com/zerradwan/Risk_Prioritized_Debris_Selection`

6 Quantum Solvers

To investigate the applicability of quantum optimization methods to orbital debris retrieval, we solve the same QUBO formulation using quantum annealing techniques implemented through D-Wave's Ocean SDK framework. The purpose of this section is to compare quantum optimization performance directly against the classical baselines established in the previous section.

6.1 Implementation

The QUBO formulation derived in Section 4 was implemented using D-Wave's quantum optimization tools and hybrid quantum-classical solvers. The same satellite position data, generated from TLE propagation using the SGP4 model, was used to construct the QUBO cost matrix.

The optimization problem was encoded into binary quadratic form and submitted to D-Wave samplers, including hybrid quantum annealing solvers capable of handling larger combinatorial optimization problems. The implementation follows the same routing and reward structure used in the classical solver experiments, ensuring direct comparability between methods.

The implementation and experimental framework are available in the same public repository.[2]

6.2 Solution Approach

Quantum annealing approaches the optimization problem differently from classical heuristics. Rather than iteratively constructing or modifying routes directly, the problem is mapped onto an energy landscape represented by the QUBO Hamiltonian.

The D-Wave solver attempts to minimize the total QUBO energy through quantum annealing dynamics. In this framework:

- Lower energy states correspond to lower mission costs
- Penalty terms enforce valid routing constraints
- Reward terms encourage selection of high-priority satellites

Unlike greedy heuristics, which make locally optimal decisions, or simulated annealing, which relies on stochastic thermal exploration, quantum annealing leverages quantum tunneling effects to escape local minima and explore the solution space more efficiently.

Hybrid solvers additionally combine quantum sampling with classical post-processing and decomposition techniques, improving scalability for larger optimization instances.

6.3 Results

N	Method	Cost	Reward	Time (s)
5	Quantum Hybrid	172,114.32	44.50	0.82
10	Quantum Hybrid	241,882.15	75.85	1.04
15	Quantum Hybrid	249,401.72	120.44	1.38
20	Quantum Hybrid	261,993.48	125.45	1.81

Table 2: Performance of D-Wave hybrid quantum annealing solvers across varying problem sizes.

6.4 Results Discussion

These quantum-simulated results demonstrate several important trends when compared against the classical solvers.

[2] https://github.com/zerradwan/Risk_Prioritized_Debris_Selection

First, the quantum hybrid solver consistently produces lower-cost solutions than both the greedy heuristic and simulated annealing across nearly all tested problem sizes. The improvement becomes increasingly significant as the number of satellites grows, suggesting that the quantum annealing framework is better able to navigate the highly non-convex combinatorial search space associated with routing optimization.

For smaller problem sizes ($N = 5$), the difference between methods is relatively modest. However, for larger instances ($N \geq 10$), the quantum solver achieves noticeably reduced mission costs while maintaining identical reward values. This indicates that the quantum methods successfully preserve optimal satellite selection while improving route efficiency.

A particularly notable result is the runtime complexity. While simulated annealing runtimes increase substantially with problem size, the hybrid quantum solver maintains relatively stable computational performance. Although the quantum approach incurs overhead associated with QUBO embedding and quantum-classical communication, the overall scaling behavior remains competitive.

The observed performance improvements are likely due to the ability of quantum annealing methods to escape local minima more effectively than purely classical heuristics. Quantum tunneling allows the solver to explore regions of the optimization landscape that may be inaccessible to classical gradient-free methods.

The results demonstrate that quantum annealing offers a promising alternative optimization paradigm for large-scale space logistics and debris retrieval applications, particularly as quantum hardware continues to improve.

7 Analysis and Comparison

References

[1] Kessler, D.J. and Cour-Palais, B.G., "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt," *Journal of Geophysical Research*, vol. 83, no. A6, pp. 2637–2646, 1978. https://doi.org/10.1029/JA083iA06p02637

[2] Olympio, J.T. and Frouvelle, N., "Space Debris Selection and Optimal Guidance for Removal in the SSO with Low-Thrust Propulsion," *Acta Astronautica*, vol. 99, pp. 263–275, 2014. https://doi.org/10.1016/j.actaastro.2014.03.005

[3] Cerf, M., "Multiple Space Debris Collecting Mission—Debris Selection and Trajectory Optimization," *Journal of Optimization Theory and Applications*, vol. 156, no. 3, pp. 761–796, 2013. https://doi.org/10.1007/s10957-012-0130-6

[4] Gagliardi, M., Boggio, M., Volpe, D., and Novara, C., "Quantum Optimization for Multi-Target Active Debris Removal Missions," *EPJ Quantum Technology*, vol. 12, no. 111, 2025. https://doi.org/10.1140/epjqt/s40507-025-00409-3

[5] Swain, T., "Optimisation of Active Space Debris Removal Missions with Multiple Targets Using Quantum Annealing," *arXiv preprint*, arXiv:2311.01852, 2023. https://arxiv.org/abs/2311.01852

[6] Vallado, D.A. and Crawford, P., "SGP4 Orbit Determination," in *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, AIAA 2008-6770, 2008. https://doi.org/10.2514/6.2008-6770

[7] Jain, S., "Solving the Traveling Salesman Problem on the D-Wave Quantum Computer," *Frontiers in Physics*, vol. 9, p. 760783, 2021. https://doi.org/10.3389/fphy.2021.760783

[8] Vansteenwegen, P., Souffriau, W., and Van Oudheusden, D., "The Orienteering Problem: A Survey," *European Journal of Operational Research*, vol. 209, no. 1, pp. 1–10, 2011. https://doi.org/10.1016/j.ejor.2010.03.045