Problem Chosen	2026 MCM/ICM Summary Sheet	Team Control Number
D		2631725

Summary

English Premier League clubs face a highly asymmetric risk environment: small performance gains deliver modest rewards, while relegation results in severe and immediate financial losses. This paper outlines a strategic decision framework that quantifies how individual players influence both on-field performance and off-field financial risk, with particular emphasis on relegation probability. Drawing on player valuation concepts such as WAR in baseball and plus-minus models in basketball, we construct a position-specific football performance metric and extend it to explicitly account for player availability and injury risk, a dimension largely ignored in existing football analytics. This allows player contributions to be evaluated not only by quality, but by reliability over the season. We put these player valuations into a team-level model that understands the nonlinear relationship between league position and revenue in the EPL, where avoiding relegation dominates all other financial considerations. Applied to Crystal Palace F.C.'s 2026–27 season, the framework links player transfers, managerial decisions, and injury insurance strategies to both expected league outcomes and downside financial risk. By translating individual player value into changes in relegation probability, the model provides actionable guidance on when squad stability is optimal and when intervention becomes economically justified.

Model Structure

Expected Player Performance and Availability Model: This portion is divided into two components: expected point contribution per player and expected games missed due to injury. By jointly accounting for performance and availability, the model provides a realistic valuation of each player's contribution to team success. These outputs are used to assess fair wage levels and to identify players who may be overvalued or undervalued relative to their expected impact.

Ticket Pricing Optimization: The ticket pricing model specifies match-level attendance as a function of ticket price, baseline demand, and stadium capacity. Demand is assumed to be price-sensitive, with attendance capped by a binding capacity constraint. Match revenue is defined as price times realized attendance, and optimal prices are obtained by solving a constrained revenue-maximization problem. Match-specific interest shifts allow prices to vary across fixtures while maintaining a consistent season-wide pricing strategy.

Revenue model: This model breaks revenue into the components listed on most premier league club statements and models each individually. Match day revenue, broadcast revenue, and commercial revenue are modeled as linear equations and are estimated using OLS, which resulted in economically viable regression coefficients given the modeling assumptions made.

Dynamic Decisions Framework: The model integrates outputs from the player valuation, league position, ticket pricing, and revenue models with an evolving probability of relegation. At each decision point, the framework evaluates potential actions by weighing their expected costs against the expected losses associated with relegation, accounting for both the likelihood and financial impact of relegation. This structure allows Crystal Palace to identify which strategic interventions are justified as risk-mitigation measures rather than reactive decisions.

Contents

1 Introduction **2**
1.1 Context & Real World Motivation . . . 2

2 Expected Player Performance and Availability Model **3**
2.1 Parameters & Notation . . . 3
2.2 Expected Point Contribution per Player . . . 3
2.3 Injury Risk Modeling . . . 5
2.4 Crystal Palace's Squad as of 2024-25 Season . . . 6

3 Ticket Pricing Optimization **9**
3.1 Model Overview . . . 9
3.2 Demand Index Construction . . . 9
3.3 Ticket Demand Specification . . . 9
3.4 Season-Level Pricing Objective . . . 10
3.5 Optimal Pricing Rule . . . 10
3.6 Sensitivity Analysis . . . 11

4 Revenue Model **13**
4.1 Parameters & Considerations . . . 13
4.2 Revenue Model . . . 13

5 Dynamic Strategic Recommendation Model **16**
5.1 Parameters & Considerations . . . 16
5.2 Classification Process . . . 17
5.3 Strategic Recommendations . . . 18
5.4 Analyses of Possible Decisions . . . 18

6 Conclusions **21**
6.1 Evaluation & Limitations . . . 21

7 Letter **23**

8 Citations **24**

9 AI Usage **25**

1. Introduction

1.1 Context & Real World Motivation

The English Premier League:
The English Premier League (EPL) is one of the largest football leagues in the world, having a potential audience of 4.7 billion people and reaching 212 territories annually (Wikipedia, 2026).

As a result football clubs bring in large amounts of revenue and compete for both financial incentives and prestige annually in the Premier league. To a large extent, Premier league clubs are a strong part of the identity of many communities inside and outside of the UK, which brings up an important consideration for many clubs. What aspect of their club should they prioritize? On the one hand, as large financial institutions, maximizing revenue is a major consideration for shareholders. However, on the other hand, maximizing performance appeases your fan base, generates popularity, and progresses you into tournaments such as the UEFA Champions league, which has prize pools and is viewed globally. However, these two objectives are often intertwined and the optimal decision for a club is often nuanced, requiring careful consideration into deciding what should be pursued. For example, improving performance often requires purchasing players from other clubs, which can often be extremely expensive, sometimes costing over £100 million(Transfermarkt,2026). This is an example of one of the many ways that club revenue and performance interact. The aim of this paper is to aid decision makers in these clubs with this balance by creating mathematical models that can quantify these considerations.

Crystal Palace:
The team we have chosen to model is Crystal Palace, a team that may be familiar due to its use as the inspiration for the acclaimed TV Series 'Ted Lasso'. Crystal Palace is a consistent middle of the league team that constantly has the potential for success, but also harbors the danger of relegation. We decided that this was the best team to model because we believe that decisions at this level are most impactful and can significantly change the trajectory of the club, but also one of our team members is a die-hard Crystal Palace fan so we did it as an act of service to him.

2. Expected Player Performance and Availability Model

2.1 Parameters & Notation

Global Notation
i - player index
P - Position group (ATT, MID, DEF, GK)
M_i - Minutes played
z(x) - z-score of variable x within position
$PP90_i$ - performance score per 90 minutes
V_{add} - value added by player vs baseline metric

Attacking Metrics
xG_{90} - Expected goals per 90 minutes
xA_{90} - Chance creation
Sh_{90} - Shots taken per 90 minutes
SoT_{90} - Shots on target per 90 minutes
SCA_{90} - Shot creating actions per 90 minutes
$ProgC_{90}$ - Progressive carries per 90 minutes

Injury Risk Availability Variables
A_i - Player age
M_i - Minutes played last season
I_i - Number of injuries last season
G_i^{miss} - Games missed last season
R_i - Recurrent injury indicator (0/1)
Pos_i - Position group (ATT/MID/DEF/GK)

2.2 Expected Point Contribution per Player

The goal of this section is to value players in the attacking position in terms of their performance compared to a benchmark. The selection of this benchmark is important to note, we considered several different options from bench players in the Crystal Palace squad to taking average across the entire premier league. In the end we decided to take all the data across the league and section them off into attackers, defenders, midfielders and goalkeepers. Then taking the 25th percentile to be a benchmark (roughly the average bench player) and convert our raw scores into z scores so that different metrics can be compared regardless of their relative frequencies, eg. many more passes than goals but goals are much more impactful than a singular pass. The z scores are then weighted and combined to give us the $PP90_i$ for each player in their respective position. The data on the premier league was collected from ”understat.com” and ”fbref.com”. Below is an outline of the data collection and z score calculations.

First find mean and standard deviation:

$$\mu = \frac{\sum_{i=0}^{n} x_i}{n}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2}$$

Now calculate associated z scores:

$$z(x) = \frac{x_i - \mu}{\sigma}$$

We plotted the distributions of expected goals and expected assists for every player in the premier league to give us the following two graphs. The rough Normal distributions confirms that we are working with data that can be approximated as normal and that using z-values is a valuable tool for this data.

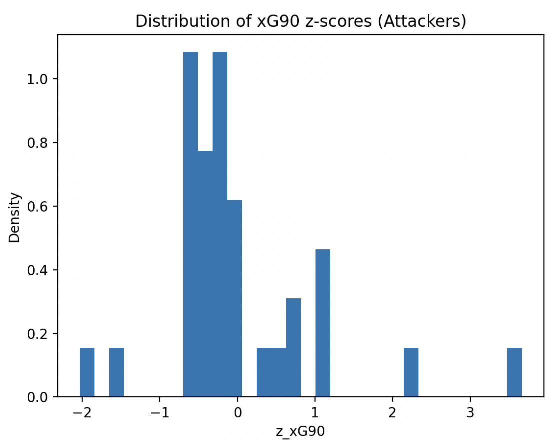

Figure 2.1: Distribution of expected goals per 90 minutes (xG/90) for attackers, *Data gathered from FBref.com*

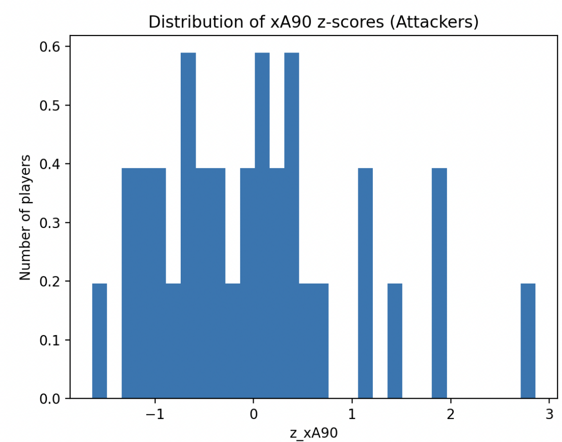

Figure 2.2: Distribution of expected assists per 90 minutes (xA/90) for attackers, *Data gathered from FBref.com*

Given the data and distributions seen you can now take a linear combination of the four stats that we are measuring and associating with attacking player value. The resulting value is the player performance score per 90 minutes. We will continue to refer to this important statistic and calculate it for each player in the Crystal Palace squad to calculate their individual value added. The formula for player performance per 90 minutes is:

$$PP90_i^{ATT} = w_1 z(xG_{90}) + w_2 z(xA_{90}) + w_3 z(SCA_{90}) + w_4 z(ProgC_{90})$$

$$PP90_i^{MID} = w_1 z(xA_{90}) + w_2 z(SCA_{90}) + w_3 z(ProgP_{90,i}) + w_4 z(ProgC_{90})$$

$$PP90_{\text{DEF}} = w_1\, z(\text{TklW90}) + w_2\, z(\text{Int90})$$

The formula for finding value added per player is:

$$V_{add} = (PP90_i - PP90_{BL}) \times M_{90,i}$$

The value for the statistic $M_{90,i}$ is calculated in the next section where we will be modeling player injury risk frequency as well as severity to give us an expected games missed/availability in minutes across the season.

2.3 Injury Risk Modeling

As stated at the end of the last section our hopes here are to quantify the likelihood of player injuries + severity to give us an estimation of how much of a season a player is likely to play. This is a crucial metric since incredibly talented players are usless to the team on the bench, modeling this significant risk greatly increases the accuracy of our estimates of player, team and financial performance over the season.

In order to get an estimate for how likely a player is to be injured we need to gather relevant data from recent seasons and see how often they were injured/whether they are likely to have recurrent injuries. The formula for injury risk we came up with is:

$$Z_i = \gamma_0 + \gamma_1 A_i + \gamma_2 \frac{M_i}{3000} + \gamma_3 R_i$$

The gammas here represent the weightings of various factors that can make a player more susceptible to injury.
γ_0 - Baseline injury risk representing the inherent probability of injury when all other factors are zero
γ_1 - Effect of player age on injury risk, measuring how injury likelihood changes with each additional year
γ_2 - Effect of minutes played on injury risk, capturing fatigue and workload accumulation
γ_3 - Effect of past injuries, measuring the contribution of injury history to future injury risk

The weightings were calculated using a logistic regression. Each gamma coefficient represents a contribution to the log-odds probability of injury Z_i. Logistic regression is appropriate here since we are modeling a binary outcome, where a player is either injured or not injured. This approach allows us to quantify the relative importance of different factors in determining a player's injury risk in a clear and interpretable way. The exact function used is:

$$\hat{\gamma} = \arg\max_{\gamma} \ \ell(\gamma) - \lambda \|\gamma\|_2^2.$$

The log likelihood term is:

$$\ell(\gamma) = \sum_{i=1}^{n} \left[y_i \log(p_i) + (1 - y_i) \log(1 - p_i) \right]$$

so p_i is a nonlinear function of γ, set the gradient of the function l(γ) to 0 and we can maximize $\ell(\gamma)$ using numerical methods.
Newton-Raphson Method:

$$\gamma^{(t+1)} = \gamma^{(t)} - H\left(\gamma^{(t)}\right)^{-1} \ g\left(\gamma^{(t)}\right)$$

$$g(\gamma) = \nabla_\gamma \ell(\gamma), \qquad H(\gamma) = \nabla_\gamma^2 \ell(\gamma)$$

To find probability of injury we take our log-odd Z_i values and use a sigmoid function to give us the probability of injury:

$$Pr(injury) = \frac{1}{1+e^{-Z_i}}$$

Given these probabilities of injury we can model the expected missed games due to injuries if we assume injuries to be independent and arrive randomly than we can model injuries as a Poisson distribution:

$$X \sim Poisson(\lambda_i)$$

Now we need to calculate λ the expected number of injuries through the relationship:

$$Pr(injury) = 1 - e^{-\lambda_i}$$

$$\lambda_i = \ln\left(1 + e^{Z_i}\right)$$

So we know the expected games missed λ. Each injury results in $S_{i,j}$ games missed where i is the player and j represents the j^{th} injury this season. Therefore games missed:

$$G_i = \sum_j S_{i,j}$$

$S_{i,j}$ are iid with mean $\mu_{S,i}$ therefore we can calculate our expected games missed:

$$E[G_i] = E[N_i] \times E[S_{i,j}] = \lambda_i \mu_{i,j}$$

We are able to calculate a value for each individual players expected games missed without needing to know the exact distributions of each players injury severity $\Sigma_{i,j}$ which each differ, instead we took data from the last two seasons coupled as well as injury history and medical guidance on average recovery times due to injury to find estimations for the $\mu_{i,j}$ of each player.
This value of expected games can be calculated for each player in the Crystal Palace squad and then applied to our value add equation from section 2.2. In the next section we recorded our own attempt at applying this theory to help gather value add scores for every individual player which we then used to help predict expected success of Crystal Palace this coming season.

2.4 Crystal Palace's Squad as of 2024-25 Season

In this section we will apply all of the statistics to the current Crystal Palace squad to give us accurate values for both player performance, injury risk, expected games missed and value added. It is important to note PP90 values are reported only for players with complete per-90 performance data across the metrics required for the position-specific model. Players lacking sufficient statistical coverage, such as Daichi Kamada, are included in the squad lists but are not assigned PP90 values to avoid introducing noise from incomplete data. Below is the values recorded of player performance:

Table 2.1: Crystal Palace — Attackers, Midfielders, Defenders, Goalkeepers (Player, Pos, Minutes, PP90, $M_{90,i}$, V_{add}). *Data gathered from FBref.com*

Player	Pos	Min	PP90	$M_{90,i}$	V_{add}
Jean-Philippe Mateta	ATT	1938	0.11	33.90	3.73
Ismaïla Sarr	ATT	1112	-0.11	34.70	-3.82
Eberechi Eze	ATT	1076	0.03	34.89	1.05

Continued on next page

Table 2.1 – continued from previous page

Player	Pos	Min	PP90	$M_{90,i}$	V_{add}
Jesurun Rak-Sakyi	ATT	0	–	36.25	–
Joél Drakes-Thomas	ATT	130	–	36.25	–
Adam Wharton	MID	1766	0.09	35.77	3.22
Jefferson Lerma	MID	903	-0.25	34.70	-8.68
Yéremy Pino	MID	1513	0.50	34.89	17.45
Daichi Kamada	MID	1195	–	34.61	–
Will Hughes	MID	966	-0.16	34.74	-5.56
Christantus Uche	MID	704	–	35.77	–
Kaden Rodney	MID	90	–	36.25	–
Chris Richards	DF	1701	0.90	34.71	31.24
Jaydee Canvot	DF	360	0.83	35.99	29.87
Maxence Lacroix	DF	2070	0.47	34.21	16.08
Marc Guéhi	DF	1800	0.12	34.50	4.14
Tyrick Mitchell	DF	2070	–	34.09	–
Nathaniel Clyne	DF	693	–	34.71	–
Daniel Muñoz	DF	1350	–	33.85	–
Dean Henderson	GK	2070	0.18	34.21	6.16
Walter Benítez	GK	0	–	35.49	–
Remi Matthews	GK	0	–	35.41	–

Weights: $PP90_i^{ATT}$

$$xG90 \rightarrow 0.40$$
$$xA90 \rightarrow 0.30$$
$$SCA90 \rightarrow 0.20$$
$$ProgC90 \rightarrow 0.10$$

$$PP90_{\text{ATT}} = 0.40\,z(xG90) + 0.30\,z(xA90) + 0.20\,z(SCA90) + 0.10\,z(ProgC90)$$

Weights:$PP90_i^{MID}$

$$xA90 \rightarrow 0.30$$
$$ProgP90 \rightarrow 0.30$$
$$Tkl{+}Int90 \rightarrow 0.25$$
$$SCA90 \rightarrow 0.15$$

$$PP90_{\text{MID}} = 0.30\,z(xA90) + 0.30\,z(ProgP90) + 0.25\,z(Tkl{+}Int90) + 0.15\,z(SCA90)$$

Weights:$PP90_i^{DEF}$

$$TklW90 \rightarrow 0.55$$
$$Int90 \rightarrow 0.45$$

$$PP90_{\text{DEF}} = 0.55\,z(TklW90) + 0.45\,z(Int90)$$

Weights: $PP90_i^{GK}$

$$Saves90 \rightarrow 0.30$$
$$Save\% \rightarrow 0.25$$
$$-GA90 \rightarrow 0.25$$
$$CS90 \rightarrow 0.20$$

$$PP90_{\text{GK}} = 0.30\,z(Saves90) + 0.25\,z(Save\%) + 0.25\,z(-GA90) + 0.20\,z(CS90)$$

All metrics are normalized by 3000 to reflect premier league theoretical maximums. Assumptions were made that season to season league standard and goals scored does not vary by much, allowing us to use data from the 2024-25 season to help calculate means and standard deviations since more data is available in a complete season. Moreover, the weightings we chose were are intentionally not derived from a purely mechanical optimization procedure, instead we made decisions based off of personal judgment as well as research papers such as (Mead, James, et al. 2023), (Goodman, Mike. 2018), and (Hewitt, Karakus. 2023).

These statistics of player value added will become essential in our strategic recommendations and team performance model when we decide on which players to protect and try to retain and which players are likely more of a hindrance to the team to keep. Quantifying a players effective value and use to a team is notoriously difficult and one of the issues we struggled with the most when trying to predict future success, especially since players are known to have break out seasons or vary a lot from past performances, there is no guarantee that the past indicates the future but we are confident that our model should provide some statistical edge/advantage for Crystal Palace, allowing them to make valuable and informed decisions.

3. Ticket Pricing Optimization

3.1 Model Overview

The objective of this section is to provide a model that determines an optimal ticket pricing strategy for the entire season that balances ticket revenue and game attendance. Rather than optimizing prices independently for each game, we seek a unified pricing rule that reflects season-long demand patterns while allowing higher prices for exceptionally popular games.

Key Assumptions:

1. The team is the sole seller of tickets for each home game.
2. Game demand depends on observable factors summarized by a demand index I_g.
3. Expected demand is decreasing in price and takes the form $Q_g(p_g) = Se^{I_g - \alpha p_g}$.
4. Attendance cannot exceed stadium capacity: $Q_g(p_g) \leq S$.
5. Management values both revenue and attendance via a weight $\lambda \geq 0$.

3.2 Demand Index Construction

To summarize the many factors affecting ticket demand, we define a game-specific demand index which aggregates team popularity, opponent attractiveness, timing effects, and market size as

$$I_g = \theta_0 + \sum_{k=1}^{K} \theta_k X_{gk}. \tag{3.1}$$

The variables X_{gk} represent observable characteristics of game g that affect ticket demand, such as opponent popularity, team performance, timing, and market size. The coefficients measure the relative influence of each factor on fan interest, with larger values indicating stronger effects on demand. Together, the weighted terms aggregate multiple demand drivers into a single, comparable index of game attractiveness.

3.3 Ticket Demand Specification

We then model the ticket demand function using a log-linear demand function,

$$Q_g(p_g) = S\, e^{I_g - \alpha p_g}, \qquad \alpha > 0. \tag{3.2}$$

where $Q_g(p_g)$ is the expected attendance for game g at price p_g, S is the maximum capacity for the stadium, I_g shifts the demand curve up and down, and α is the common price sensitivity across the season, where

$$\alpha = -\frac{\partial \ln Q_g(p_g)}{\partial p_g}. \tag{3.3}$$

3.4 Season-Level Pricing Objective

If the team only maximizes ticket revenue, the season objective is

$$\max_{\{p_g\}_{g=1}^{G}} \sum_{g=1}^{G} p_g\, Q_g(p_g). \tag{3.4}$$

To account for long-run value created by attendance (e.g., converting some attendees into season-ticket holders), we assign a constant expected additional value λ per attendee. The season objective becomes

$$\max_{\{p_g\}_{g=1}^{G}} \sum_{g=1}^{G} \Big(p_g\, Q_g(p_g) + \lambda\, Q_g(p_g) \Big). \tag{3.5}$$

We interpret λ as the expected future value per attendee:

$$\lambda \;=\; (\text{conversion probability}) \times (\text{net value of one season ticket}).$$

For example, if the conversion probability is 0.05 and the net value of one season ticket is \$400, then

$$\lambda = 0.05 \times 400 = 20.$$

3.5 Optimal Pricing Rule

The objective contribution of game g is

$$(p_g + \lambda)\, Q_g(p_g) = (p_g + \lambda)\, S e^{I_g - \alpha p_g}.$$

Maximizing with respect to p_g yields the first-order condition

$$\frac{d}{dp_g}\Big[(p_g + \lambda)\, S e^{I_g - \alpha p_g} \Big] = 0.$$

implying the unconstrained optimal price

$$p_g^* = \frac{1}{\alpha} - \lambda.$$

However, attendance cannot exceed stadium capacity S. Thus, prices must also satisfy

$$Q_g(p_g) \le S.$$

Using $Q_g(p_g) = Se^{I_g - \alpha p_g}$, the capacity constraint binds when

$$Se^{I_g - \alpha p_g} = S,$$

which implies

$$p_g = \frac{I_g}{\alpha}.$$

Therefore, the optimal price for game g is

$$p_g^* = \max\left(\frac{1}{\alpha} - \lambda,\ \frac{I_g}{\alpha}\right), \tag{3.6}$$

where the max operator reflects a standard Kuhn–Tucker outcome in constrained optimization, where either the unconstrained revenue optimum holds or the capacity constraint binds, depending on the level of game demand.
This model outputs a single season-wide strategy based on season-level fixed parameters, while still generating game-by-game pricing decisions. Since we use the maximum operator, the model directly reports the final game-by-game prices as most games are priced at the season base level, while only high-demand games with large I_g generate higher prices. Using fixed season-level parameters ensures that ticket pricing reflects a consistent and deliberate management strategy rather than short-term, ad hoc reactions. This structure also enables straightforward scenario analysis, as management can adjust a small number of parameters to evaluate how different strategic priorities affect prices across the entire season. At the same time, the model is operationally realistic to implement and is also computationally simple avoiding complex dynamic programming or repeated recalibration.

3.6 Sensitivity Analysis

Table 3.1 shows how uncertainty in the two key season-level parameters, α and λ, affects the model's recommended ticket prices, p^*. This sensitivity analysis demonstrates that the model remains interpretable and robust across plausible parameter ranges, even when precise historical ticket prices are unavailable online. The ranges for α and λ are chosen to reflect economically plausible values rather than precise estimates. In particular, α spans a range of price sensitivities consistent with inelastic demand for matchday tickets, while λ covers strategies ranging from pure revenue maximization to moderate emphasis on attendance and long-run fan value. For Crystal Palace, using a balanced strategy with $\lambda = 5$, a standard game attractiveness of $I_g = 0.6$ (with $I_g \in [0, 1.5]$), and a representative price sensitivity of $\alpha = 0.02$, the model yields an optimal ticket price of £45. This value is close to the current average adult ticket price of £48 for the upcoming home match against Burnley on 02/11/2026, providing external validation of the model's realism (CPFC Ticketing Website, 2026).

Table 3.1: Sensitivity of optimal ticket price $p_g^* = \max\left(\frac{1}{\alpha} - \lambda, \frac{I_g}{\alpha}\right)$ for three demand levels I_g. Rows vary α and columns vary λ.

When $I_g = 0.6$

$\alpha \setminus \lambda$	0	2.5	5	7.5	10
0.01	100.0	97.5	95.0	92.5	90.0
0.02	50.0	47.5	45.0	42.5	40.0
0.03	33.33	30.83	28.33	25.83	23.33
0.04	25.0	22.5	20.0	17.5	15.0
0.05	20.0	17.5	15.0	12.5	12.0

When $I_g = 0.9$

$\alpha \setminus \lambda$	0	2.5	5	7.5	10
0.01	100.0	97.5	95.0	92.5	90.0
0.02	50.0	47.5	45.0	45.0	45.0
0.03	33.33	30.83	30.0	30.0	30.0
0.04	25.0	22.5	22.5	22.5	22.5
0.05	20.0	18.0	18.0	18.0	18.0

When $I_g = 1.2$

$\alpha \setminus \lambda$	0	2.5	5	7.5	10
0.01	120.0	120.0	120.0	120.0	120.0
0.02	60.0	60.0	60.0	60.0	60.0
0.03	40.0	40.0	40.0	40.0	40.0
0.04	30.0	30.0	30.0	30.0	30.0
0.05	24.0	24.0	24.0	24.0	24.0

If the club has historical match-level ticketing records (which it would, via its ticketing system), the price sensitivity parameter α can be estimated directly rather than assumed. Starting from the demand specification and taking logs and rearranging, we obtain

$$\begin{aligned} \ln\left(\frac{Q_g}{S}\right) &= I_g - \alpha p_g, \\ \ln\left(\frac{Q_g}{S}\right) &= \beta^\top X_g - \alpha p_g + \varepsilon_g, \end{aligned} \tag{3.7}$$

where X_g is a vector of observable game characteristics (e.g., opponent strength, timing, rivalry indicators) that shift ticket demand independently of price and together form the demand index, I_g. Calibrating α will allow for even more quantitatively meaningful observations to be made. However, historical Crystal Palace ticket price data is extremely scarce online and we currently lack the scale to be able to accurately estimate α.

4. Revenue Model

4.1 Parameters & Considerations

Global Notation
R_t = Revenue(Season)
M_t = Match day Revenue(Season)
B_t = Broadcast Revenue(Season)
C_t = Commercial Revenue(Season)
Cap = Stadium Capacity
Att_t = Attendance(Season)
$LPos_t$ = League position
$G^{\mathbf{club}_{start}}$ = club Google searches at the beginning of the season
$G^{\mathbf{club}_{end}}$ = club Google searches at the end of the season
Att = Attendance(season)
$\mathrm{G_t}_t$ = Google Search interest(season)

4.2 Revenue Model

The aim of this section is to model Crystal Palace's revenue effectively in order to be able to be used as a viable input in the dynamic decision making model. We have decided to break total revenue into its major components:

$$R_t = M_t + B_t + C_t$$

In club financial reports, total revenue is reported as the sum of these three components, thus we have decided to model total revenue by modeling each of the individual components where each component is modeled by a linear regression as follows:

$$\mathrm{M}_t = 19 \cdot Cap_t \cdot U_t \cdot Yield_{t-1}$$

In order to model match day revenue effectively, we have to model what drives attendance in a given game. Given that Selhurst Park is a relatively small stadium, capacity is often the limiting factor in attendance, so we decided to model stadium utilization to account for this as follows:

$$\mathrm{U}_t = \frac{Att_t}{Cap}$$

$$\mathrm{Yield}_{t-1} = \frac{M_{t-1}}{Att_{t-1}}$$

Due to the consistent near capacity attendance of Selhurst Park, match day revenue isn't a result of absolute fan base size, but rather shifts in demand across seasons. We have therefore modeled relative

[0]Unless otherwise stated, all monetary values are reported in thousands of pounds sterling (£000).

change in club popularity via Google trends data and since sellout rates are consistently high, shifts in demand are more informative:

$$\mathrm{Pop}_t = \Delta \log(\mathrm{G}_t)$$

Pop_t is standardized using the standardization formula:

$$\widetilde{X}_t = \frac{X_t - \mu_X}{\sigma_X}$$

And Utilization is modeled by the regression with the standardized variable:

$$U_t = \alpha_0 + \alpha_1 \widetilde{\mathrm{Pop}}_t + \varepsilon_t$$

$$U_t = \min\left\{1,\ \alpha_0 + \alpha_1 \widetilde{\mathrm{Pop}}_t + \varepsilon_t\right\}$$

α_0 represents the baseline expected stadium utilization in a season given average club popularity and game intensity.
α_1 measures the change in season-level stadium utilization associated with a one standard deviation increase in club popularity growth
ε_t represents the error term that signifies changes in utilization not linked to either of the above variables

Next, the broadcast revenue function is more simply modeled as broadcast revenue tends to be rule based, and is not influenced by fan demand variables and is modeled as follows:

$$P_t = 21 - \mathrm{Pos}_t$$

where Pos_t is the final league position in season t

$$B_t = \beta_0 + \beta_1 P_t + \beta_2 t + \eta_t$$

where,
t is a season index capturing league-wide media rights growth,
η_t is an idiosyncratic error term.

β_0 captures the baseline equal-share component of broadcast revenue,
β_1 captures the marginal gain in revenue based on league performance merit.
β_2 captures exogenous growth caused by league media rights cycles.
η_t contains broadcast revenue variation that stem from non systematic fluctuations in revenue, including, tournament placements prizes and one off televised events.

Finally, we model the Commercial Revenue. Since most commercial revenue comes from multi year brand deals and contracts, it is inherently sticky and adjusts slowly with time. Thus we have chosen to model Commercial revenue log-level auto regressive process with a time trend to capture the structural growth, whilst incorporating an error term that captures large shocks that may be driven by sudden sponsorship changes or tournament exposure.

$$c_t = \log(C_t)$$

$$c_t = \gamma_0 + \rho\, c_{t-1} + \gamma_1\, t + \xi_t$$

$$C_t = \exp(\gamma_0 + \rho\ \log(C_{t-1}) + \gamma_1 t + \xi_t)$$

where,
t is a deterministic season index
ρ captures persistence in commercial revenue as a result of multi year contracts
γ_1 is captures exogenous drift such as inflation and structural growth
ξ_t is composed of non systematic, one off, instances such as tournament exposure and sponsorship changes.

To obtain the various weights, we used ordinary least squares regressions. Due to limited availability of club level data, OLS is used as a directional indicator in the regressions to observe directional sensitivities instead of precise causal effects. Regardless, the coefficients provide economically interoperable estimates which are as follows:

Table 4.1: Estimated Revenue Model Parameters

Component	**Parameter**	**Estimate**
Matchday (M_t)	α_0	1.07597
	α_1	0.05357
Broadcast (B_t)	β_0	49,581.99
	β_1	6,309.21
	β_2	3,730.31
Commercial (C_t)	γ_0	9.10874
	ρ	0.11435
	γ_1	0.02827

The data to run the regressions was collected from (KinnAIrd Intelligence, 2025), (European-Football-Statistics, 2025), and (Google Trends, 2026) The regressions have outputted economically viable coefficients that can be used as decent indicators of the trajectory of Revenue based on the models inputs. One coefficient worth mentioning is ρ as it is relatively low, which implies that previous commercial revenue has a limited influence on this season. This weakens the thesis that commercial revenue is unresponsive to changes; however, it can be partially explained by limited data and post Covid revenue effects.
Using the model to estimate revenue for the 25/26 season we get an estimate of 156.4 million, which is a reasonable indicator given Crystal Palace's lower 24/25 position finish.

5. Dynamic Strategic Recommendation Model

5.1 Parameters & Considerations

This section will define the required data and inputs necessary for our model to determine the club's strategic state at each decision point and be able to provide valid and informed recommendations. These parameters summarize on-field performance, financial health, risk and health exposure. These will collectively form the state variables upon which our mathematical model operates.

Player Level Parameters
P_i — Expected Point Contribution of player *i*
$A_i \in [0, 1]$ — Availability Factor of Player *i*
G_i — Expected Games Missed by Player *i*
W_i — Annual wage cost of player *i*

Squad Level Parameters
SS — Squad strength index

$$SS = \sum_i P_i \times A_i$$

DP — Depth Penalty
Pt_s — Expected Season Points
Pt_{safe} — Historical Safety Threshold To Avoid Relegation

Revenue & Cost Parameters
Rev — Expected total revenue
Rev_b — Broadcast revenue component
Rev_c — Commercial revenue component
Cost — Expected Total costs
W_{tot} — Total Wage Bill

$$W_{tot} = \sum_i W_i$$

5.2 Classification Process

The first thing we need to start off with is calculating the expected points at the end of the season. We can do this quickly using our player value added calculated in section 2. We'll take the average points scored by Crystal Palace in the last three seasons as out baseline α_0 and then multiply the sum of the starting XI players by a scaling factor α_1 and add it to the baseline to predict the total points by the end of the season.

$$Pts = \alpha_0 + \alpha_1 \sum_{i=0}^{11} V_{add} + \eta$$

α_0 and α_1 can be calculated by:

$$\alpha_0 = \frac{\text{Pts}_{t-1} + \text{Pts}_{t-2} + \text{Pts}_{t-3}}{3}$$

$$\alpha_1 = \frac{\text{Pts}_{\text{last season}} - \alpha_0}{\sum_{i=1}^{11} V_{\text{add},i}^{\text{last season}}}$$

η represents the error term of the points scored per season. η can be calculated by using the points from last season as well as the prediction using our method to get a sense of how accurate it has been in the past:

$$\eta_s = \text{Pts}_s - \left(\alpha_0 + \alpha_1 \sum_{i=1}^{11} \widetilde{V}_{\text{add},i,s} \right).$$

Using previous seasons data we get $\alpha_0 = 49, \alpha_1 = 0.06, \eta = -4.5$
Next given, our predicted points we want the probability of relegation, since this is one of our greatest considerations and will become a very important factor in our models strategic decision making. We need to find the cut off of points that will make us very unlikely to be relegated at the end of the season i.e . the threshold for relegation (Pts_{safe}).

$$Pts_{safe} = 38$$

Historically scoring 38 points in a season puts you in a safe position to not be relegated in the premier league. Now we want to know given our prediction what is the probability of it being below 38 points. To do this we can model our prediction as a normal distribution and calculate a probability.

$$\text{Points} \sim \mathcal{N}\left(\widehat{\text{Points}},\ 4.5^2\right).$$

The z-score is:

$$z = \frac{Pts - 38}{4.5}$$

So the probability of relegation is:

$$\text{Pr}_{rel} = \Phi\left(\frac{38 - \widehat{Pts}}{4.5}\right)$$

5.3 Strategic Recommendations

The core of our ideas for strategic recommendation is making positive strategic actions that are justified if the marginal reduction in expected relegation loss/expected improvement in end of season rewards exceeds its marginal cost.

The calculated expected loss from relegation is:

$$E[Loss] = Pr_{relR}$$

Where L_R is the financial loss due to relegation (information gathered from) valued at £100M. They stand to lose £67M in broadcasting rights alone not to mention decrease in fan attendance, ticket pricing, merchandise sales, brand deals, etc.

When calculating and evaluating the payoffs associated with player transfers and protection, it is very important for us to consider the decrease in $E[Loss]$ and use this to decide whether the cost of the player is justified. In equation terms if a decision is good:

$$L_R \Delta Pr_{rel} \geq Cost(a)$$

Where "a" represents a strategic action: managerial changes, transfers targeting weakest positions, injury insurance, selling players, etc.

5.4 Analyses of Possible Decisions

This section will have a lot of influence from our own knowledge of the premier league and Crystal Palace's recent performances. It is important to note that the club lost Eze and Olise going into this season and as a result the attack and midfield has been quite weak, this was heavily supported by the difference in PP_{90} values from the much higher defenders than attackers. Thus, we think a great place to start analyzing decisions is with the acquisition of attacking and midfield players.

Player Transfers

When selecting a good player it's important to take into account academy vs premier league vs other leagues. Moreover although players are more expensive in January we have more data on the players as well as our standing in the league and are able to make relegation predictions with higher certainty, hence we are willing to pay more for the reduction in uncertainty. We decided that we needed a robust player who is younger with a strong record of goals and assists. We ran the championship players through our same PP_{90} calculations to give us the best value add players in the team, avoiding players with blatant injury issues.

Our model identified Will Keane as a potential player for acquisition with a PP_{90} of 4.04, higher than that of Mateta the star attacker in the team currently. Now we can calculate the improvement in expected loss to see theoretically how much money we will effectively save if we are to acquire Will

Keane. The new team's expected points 57.8 as calculated from the framework outlined, a huge jump compared to the 48.9 that they are expected to achieve currently.

$$\Delta p_R = 0.008 - 0.00000758 = 0.007999242.$$

The difference in probability of relegation went from roughly 0.8% to 0.00075% virtually impossible. So our change in expected loss is roughly £800,000, the effective amount we save with Will Keane. The market value of Will Keane currently is about £500,000, making the increase in this case outweigh the cost since $800,000 > 500,000$.

$$\therefore L_R \cdot \Delta p_R \geq C_{\text{mgr}}$$

however, it is important to note again, that although the cost of relegation is extremely high, there are very minimal benefits to improving league table position near the relegation zone. For example, going from 16-15th has virtually no increase in financial benefits. To make an argument for a new signing at a point when the club is not doing great financially, we would need to say that the social media boost and possible brand deals from an exciting new signing like Will Keane would be a great benefit to the club and pay off financially, since the likelihood of relegation is already low.

Managerial Change Under Relegation Risk

Empirical evidence indicates that managerial changes for lower–table teams can generate meaningful short–term point gains. Since relegation outcomes are typically determined by narrow point margins, even modest improvements in expected points can translate into substantial reductions in p_R. Under our calibration, where approximately four points often separate survival from relegation, a gain of two to three points may be sufficient to justify the intervention.

Conversely, when relegation probability is already low or when squad quality constraints dominate performance outcomes, Δp_R^{mgr} is small and managerial continuity is optimal. For Crystal Palace, managerial change should therefore be viewed as an insurance–type intervention, undertaken only when it offers a larger marginal reduction in expected relegation loss than alternative strategies such as targeted transfers or squad depth investments.

Given the probability of relegation for Crystal Palace is 0.8% based on our estimation, it's unlikely that we will pursue a short-term transfer for a new manager. However, if the club were in the relegation zone at 38 points, we assume an associated relegation probability of $p_R = 0.50$. Studies have shown a managerial change increases the team's expected points total by 8 points over 10 games (London Economics, 2013), if we assume this and hire a new manager, our model suggests that relegation probability falls to $p_R = 0.038$. This corresponds to a reduction in relegation probability of

$$\Delta p_R = 0.50 - 0.038 = 0.462.$$

Assuming a relegation loss of $L_R = 100$ million and a managerial change cost of $C_{\text{mgr}} = 20$ million, a managerial change must reduce relegation probability by at least 20 percentage points to break even on relegation-risk grounds alone, since

$$L_R \cdot \Delta p_R \geq C_{\text{mgr}} \quad \Longrightarrow \quad \Delta p_R \geq 0.20.$$

Because the estimated reduction in relegation probability is approximately 46.2 percentage points, the expected benefit of a managerial change substantially exceeds its cost. Therefore, if Crystal Palace were in the relegation zone, the model would strongly recommend a change in manager.

Injury Insurance

Based on the current 25/26 squad of Crystal Palace, we've noticed through our player performance model a heavy reliance on defender Chris Richards with a V_{add} of 31.24. If Chris Richards gets injured, we need to provide injury insurance through player acquisition so his absence does not substantially increase the club's chance of financial relegation. First let's calculate the probability of injury and then we can find the increased probability of relegation and from there the increased expected loss. If we take a loan player, and replace the injured position, even if we were to lose one of our best players the effect of the injury on our expected points and therefore probability or relegation is mitigated so that the expected loss from the scenario outweighs the cost of the loan.

The expected probability of relegation if Chris Richards is injured jumps from 0.8% to 2.2% using our model's framework. The increase in expected loss is:

$$(0.022 - 0.008) \times 100,000,000 = 1,400,000$$

A £1,400,000 increase in expected loss is huge, which makes sense since it reflects the large impact that Chris Richards has on the team as well has his V_{add} being the largest in the team. Knowing this, we can justify loaning a player that mitigates the increase in expected loss due to relegation. This loan acts as a hedge for the team, although the player may never see the pitch and in an ideal world does not, in the case of the worst outcome, they become invaluable and worth much more than they are paid.

Figure 5.1: The Future of Palace

6. Conclusions

6.1 Evaluation & Limitations

Player Performance
Firstly, our model calculates position-specific PP_{90} variations, rather than evaluating every player on the same scale, eg. evaluating goalkeeper performance and value by expected goals is ridiculous, the model understands how different stats are more or less important based on position. Second, expressing statistics in a per 90 minute playing time reduces bias based on how many minutes each player plays. Third, the model explicitly takes into account player availability by estimating injury risk and expected games missed. This addresses a key weakness of many performance models that implicitly assume players are available for an entire season, which could be a fatal detail to overlook when advising a sports team.
However, it is important to understand the limitations of our model in order to know how to use the data effectively.
The weights used to construct the PP_{90} metrics are chosen based on football intuition and guidance from research we found rather than being estimated through a purely data-driven optimization process. While this makes the data easier to understand and avoids overfitting, it introduces subjectivity into the model.
The PP_{90} framework assumes that each performance metric contributes independently and linearly to overall player value. In practice, football performance often involves nonlinear interactions between skills, such as the complementarity between chance creation and finishing ability. This linear addition of player values fails to take into account player chemistry as well. We had plans to model player chemistry as covariance between them, but we were unable to find data of every player being subbed out with the rest of the team still intact. This data would be more readily available to the club, who could make substitutions purely to see the impact of players subbed on and off. We leave the problem of modeling chemistry between players to the reader...
The decision to use the 25th percentile of league-wide performance as a proxy for a replacement-level or bench player is a practical simplification rather than a theoretically exact choice. Bench roles vary significantly across clubs and positions, and a league-wide percentile may not perfectly capture Crystal Palace's internal replacement options.
The model assumes that injuries occur independently and that injury events arrive randomly over time, allowing them to be modeled using a Poisson process. In reality, injuries are often correlated due to factors such as fixture congestion, accumulated fatigue, and incomplete recovery from prior injuries.
Potential extensions of the model include estimating metric weights using regularized regression techniques, introducing nonlinear interactions between performance variables, modeling injury severity using survival analysis, and incorporating tactical or lineup-based effects.

Ticket Optimization
The effectiveness of the ticket pricing model depends critically on the estimation of the price sensitivity parameter, α. In our analysis, accurately estimating α was constrained by the limited availability of historical, game-level ticket pricing data, which prevented precise recovery of demand responses from

past fixtures. As a result, our baseline calibration relies on conservative, model-consistent assumptions rather than direct estimation. For the club itself, however, this limitation is largely practical rather than structural: detailed historical pricing and sales data are readily accessible internally, allowing α to be estimated with substantially greater precision using observed price–quantity relationships.

The model further assumes a downward-sloping demand curve, which is appropriate over most of the relevant price range but may not hold uniformly as attendance approaches stadium capacity. Near sell-out conditions, scarcity effects and fan expectations can lead to locally upward-sloping observed demand, where higher prices coincide with increased sales velocity. While this introduces potential nonlinearity at the upper tail of demand, the pricing framework remains robust because capacity constraints are explicitly accounted for in the optimization problem.

Despite these data and structural limitations, the model produces revenue-maximizing ticket prices that are economically reasonable and consistent with observed pricing behavior in comparable EPL fixtures. Importantly, the framework is designed to improve in line with data availability, incorporating historical ticket prices, sales timing, and match-specific demand indicators would allow for re-estimation of α, refinement of demand curvature near capacity, and tighter revenue predictions. As such, the current results should be interpreted as conservative benchmarks, with clear potential for accuracy gains as richer internal data are introduced.

To assess robustness, we conducted sensitivity checks over plausible ranges of α and demand intensity, observing that the revenue-maximizing ticket price varies smoothly rather than discontinuously across parameter values. Importantly, under all reasonable calibrations, the optimal price remains within a realistic range relative to current EPL pricing benchmarks. This stability indicates that the model's recommendations are not driven by knife-edge parameter choices, reinforcing its suitability as a decision-support tool.

Revenue

As for the assumptions of this model, the first main assumption is that match day revenue is capacity constrained, meaning that that growth is mainly from yield per attendee and stadium capacity. The second assumption is that broadcast revenue is rule based, which seems true on balance sheet breakdowns but may not account for exogenous TV deals. Lastly, we assume that sponsorship and commercial revenue is sticky. this is the weakest assumption as the low rho indicates otherwise. However, this needs to be reevaluated with more data and the rho is still an economically viable coefficient, it just is not as strong as expected.

In terms of the approximation using the weights, the revenue outputted for 25/26 is a reasonable estimate given the only major change in the data was a lower placement in the league in the previous season. However, the largest limitation in these estimated coefficients is the lack of data and the effect post Covid shocks in the sample.

Potential improvements in the commercial revenue model with more explicit potential and current sponsorship data would significantly improve the accuracy of this model. However, given our limited access to such data, we are unable to take such considerations into account, however, given the club's data this is definitely a feasible extension.

7. Letter

Dear Crystal Palace F.C,

After conducting an in-depth analysis on your squad's player value, revenue generation, and ticket pricing model, our team has developed a comprehensive strategic plan for Crystal Palace F.C. Our research indicates that Crystal Palace should avoid purchasing additional players and changing managers unless there are exogenous shocks and to look into loaning players as an injury hedge for Palace's most impactful players.

Player Transfers: Our player performance analysis suggests that, compared to the rest of the league, Crystal Palace are much stronger defensively and in midfield than in attack. That said, given the asymmetric nature of relegation risk and the club's current position in 15th place with roughly 17 games remaining and only a small buffer above the relegation zone, marginal improvements to the attacking unit are unlikely to significantly reduce relegation risk. As a result, the model does not justify aggressive attacking signings and instead supports maintaining squad continuity while managing downside risk through short-term, low-commitment options such as loans which serve as injury insurance.

Managerial Change Under Relegation Risk: Our performance and revenue framework highlights that relegation risk introduces a sharp nonlinearity in both sporting outcomes and financial returns. When the probability of relegation rises beyond a critical threshold, expected losses (through reduced broadcasting revenue, lower match day demand, and weakened commercial value) grow disproportionately. In this regime, managerial effectiveness becomes a first-order determinant of outcomes rather than a marginal factor. As a result, our model supports the view that timely managerial intervention under extremely elevated relegation risk can act as a savior for Crystal Palace as a last ditch strategy to prevent relegation.

Potential Stadium Expansion: Through the development of our revenue and ticket pricing models, it becomes clear that stadium capacity is binding for the majority of Crystal Palace home fixtures. In addition, revenue projections from our model indicate that the club will require additional avenues for sustainable revenue growth. One such avenue is stadium expansion, and we are encouraged to learn that the acquisition of external properties to facilitate expansion is already underway. This strategy would directly increase revenue capacity and provide greater flexibility within the ticket pricing framework. Our model strongly supports this investment as a coherent extension of the club's long-term revenue strategy.

Thank you for your time,
Team 2631725

8. Citations

1. Google Trends. *Explore*. https://trends.google.com/explore?q=/m/0223bl&date=now+1-d&geo=US. Accessed 2 Feb. 2026.

2. Kinnaird Sports Intelligence. *Crystal Palace Financial Dashboard*. https://kinnairdsports.com/crystal-palace. Accessed 2 Feb. 2026.

3. Mead, James, et al. "Expected Goals in Football: Improving Model Performance and Demonstrating Value." *PLOS ONE*, vol. 18, no. 4, 5 Apr. 2023, e0282295. https://doi.org/10.1371/journal.pone.0282295.

4. The Football Analyst. "Progressive Carries – Football Statistics Explained." 22 Apr. 2025. https://the-footballanalyst.com/progressive-carries-football-statistics-explained/.

5. Goodman, Mike. "A New Way to Measure Keepers' Shot Stopping: Post-Shot Expected Goals." *Hudl*, 12 Nov. 2018. https://www.hudl.com/blog/a-new-way-to-measure-keepers-shot-stopping-post-shot-expected-goals. Accessed 1 Feb. 2026.

6. Hewitt, James Henry, and Oktay Karakus. "A Machine Learning Approach for Player and Position Adjusted Expected Goals in Football (Soccer)." 19 Jan. 2023. https://doi.org/10.1016/j.fraope.2023.100034.

7. Opta Analyst. *Premier League Table*. 28 Jan. 2026. https://theanalyst.com/competition/premier-league/table.

8. Crystal Palace F.C. eTicketing. *Home Matches Events Page*. https://www.eticketing.co.uk/cpfc/Events?preFilter=5&preFilterName=2.%20Home%20Matches. Accessed 2 Feb. 2026.

9. London Economics. *Survival or relegation? The impact of a managerial change in the Premier League*. Dec. 2013. https://londoneconomics.co.uk/wp-content/uploads/2013/12/Economic-impact-of-managerial-change-in-Premier-League-Final.pdf. Accessed 2 Feb. 2026.

10. Wikipedia contributors. *Premier League*. Wikipedia. https://en.wikipedia.org/wiki/Premier_League. Accessed 2 Feb. 2026.

11. Transfermarkt. *Premier League: Market Values*. https://www.transfermarkt.com/premier-league/marktwerte/wettbewerb/GB1. Accessed 2 Feb. 2026.

9. AI Usage

ChatGPT (version 5.0) was used to assist with grammar checking, spelling, LaTeX formatting, and to help identify relevant academic literature and publicly available sources for league-wide data. We also used the same LLM to get citations for research papers and double checked with citation generators.

Additionally, Google Gemini was used to generate Figure 5.1, a conceptual illustrative image of Crystal Palace F.C. winning the league. This figure is illustrative only and does not contribute to the analytical results of the paper.